OPERATION OPTIMIZATION CONSIDERING ORDER CANCELLATION AND TICKET DISCOUNT FOR ON-DEMAND BUS SYSTEM

X CHEN

Chang'an University, Middle Section of Nan Erhuan Road, Xi'an, China
Tel:15829665058; Email: 190393924@qq.com

ABSTRACT

The emergence of multi-source and mass travel data and the gradual maturity of acquisition methods make the realization methods of travel demand more diversified. The on-demand bus is a pattern that can meet the needs of personalized and high-quality travel. This project proposes an on-demand bus system based on response to random users' real-time requests. Under the condition of fixed origin points and destination points, taking the minimum total waiting time of passengers and the maximum profit of the bus as the goals, a model based on the variables of bus fare discount and request valid time is established, and LINGO and the actual car-hailing data are used to calculate and check the model. Through calculation results and the comparison with the traditional bus and the express, the on-demand bus system has shorter travel time than the traditional bus and costs less than the express. Thus the research can be thought to provide a scientific basis for the establishment of the on-bus bus system, and provide theoretical support for further research.

Keywords: on-demand bus, waiting time of passengers, bus fare compensation mechanism

1 Introduction

With the rapid development of the economy, the level of motorization in Chinese cities has been greatly promoted. To satisfy the increasing travel demand of residents and alleviate the problem of urban traffic congestion, after the establishment of the strategic goal of bus priority development, the local government has promulgated a series of policies and measures, such as expanding the layout of the public transportation network and strengthening the construction of rail transit. These measures ease the traffic pressure in the city to a certain extent, but still can’t completely meet the residents' diversified travel demands, especially in the increasingly high requirements on life quality today. People focus more on the travel experience of minority, individualization and high-quality. Meanwhile the accuracy, accessibility, diversity and comfort of travel gradually get more attention.
With the extensive use of smart phone App based on the LBS (location based service), it is

possible to obtain the user's mobile terminal location information, which provides technical support for the rapid development of express cars, premier cars and car sharing. But the high travel costs of express cars and premier cars are unaffordable for most people. Therefore, it is imperative to develop an on-demand bus, whose travel experience is more comfortable than the traditional bus, and the travel cost is lower than the express cars. The on-demand bus is a new travel mode based on the internet background. It provides a new idea for improving the quality of the bus transit system, and it is a supplement to the low quality of the existing public transport service.

Yi Liu[1] compared the passenger attribute, line stations, bus fare, service quality and other aspects of on-demand bus and the traditional bus, established site and route planning model and ticket pricing model, laying a theoretical foundation for further on-demand bus research. Xiaojun Guo[2] analyzed the key elements of the on-demand bus system and established the on-demand bus model. His research provided the basis to the on-demand bus system in the sparsely populated area. Bin Li[3] built Logistic model of on-demand bus fare, and he studied the ticket ranges that commuters could accept, the study pointed out that whether there were seats, the number of bus stops along the way and commuting distance were important factors affecting the city residents receive an on-demand bus. Di Huang[4] proposed a new nonlinear bus fare structure based on distance, which was measured by Euclidean distance between the destination and the destination. The results indicated the advantage over existing fare structures: it reflected the 'true cost' of a passenger's trip. Jinxu Chen[5] proposed a methodology for the optimal design of a suburban bus route for airport access, with the objective of minimizing the total access time and used the artificial bee colony (ABC) approach to solve the model. He found under this method, the total travel time was less than the normal.

The previous studies focused more on the influence factors, bus fare, travel time, travel cost, etc. They all got outstanding achievements. However, they all considered the problems separately and studied little on the passengers' waiting time[6][7][8][9]. This paper focuses on the passengers' waiting time and the bus profit and puts forward a mechanism of bus fare compensation and a mechanism of reservation cancelling. Based on the two mechanisms, on the case of fixed origin points and destination points, a model aiming at the maximum profit and the minimum total waiting time is established.

2 Model establishment

2.1 Variables and parameters description

parameters	parameters description
N	bus capacity
X_0	initial bus fare
t	the time that the bus begins to accept an order
t_i	$i = 0, 1, \ldots, \beta$, each expecting departure time
t_β	the latest time for departure
T	the time interval between the two expected departure times, which is called the valid request time
α	fare discount
n_0	the number of the passengers reached between t and t_0
n_i	the number of the passengers reached between t_{i-1} and t_i
n'_{0j}	the number of the passengers reaching between t and t_0 and leaving between t_{j-1} and t_j, $j \geq 0$

n'_{ij}	the number of the passengers reaching between t_{i-1} and t_i leaving between t_{j-1} and t_j, $\mathrm{i} < \mathrm{j}$ and $\mathrm{i} \geq 1$, $\mathrm{j} \geq 2$
y	the cost of a bus running once
m	the cost of a bus running per kilometer
L	running mileage between the origin point and the destination point
m_0	the fixed cost of a bus running once, including shares of buses purchasing, personnel wages, etc.
λ	exponential distribution parameter
f(x)	exponential distribution density function
γ	the expected profit margin
ρ	the minimum guaranteed profit margin
t_{ip}	the Pth passenger arriving at $t_{i-1} \sim t_i$
$t_j - t_{ip}$	the Pth passenger arriving at $t_{i-1} \sim t_i$ when the bus leaves at t_j
$T_{\text{sum i}}$	the total waiting time of the passengers, which is the total waiting time of the passengers who are on the bus after departure (that is, the waiting time of the passengers who cancelled reservations is not taken into consideration.)
$\overline{T_i}$	The average waiting time, which is the ratio of the total waiting time to the total number of passengers on the bus after the departure
z	the income of one running bus
z_1	the profit of one running bus
k	The times of departure delaying

2.2 Model hypotheses

Some hypotheses were proposed during modelling:

1). Passengers send out requests, choose the expected boarding time and ensure that they can get on the bus before departure;

2). The running distance between any origin point and destination point in the road network is known;

3). During each travel, the probability of passengers cancelling reservations obeys the exponential distribution;

4). It is considered that the bus is running at a constant speed in the road network, not considering the influence of traffic conditions on the speed;

5). The model is based on the operational characteristics of the mature on-demand transit service market;

6). The waiting time of the passengers who cancelled the reservation during the waiting period is not included in the total waiting time;

7). Only the benefits of single bus was taken into account;

8). The passengers who send an order between t_i and t_{i+1} can only cancel the order after t_{i+1};

9). When receiving N orders, the system will not accept the extra orders, and the extra orders will be assigned to the next bus.

2.3 Variables of the model

α — the bus fare discount per extension of waiting time, $0 < \alpha < 1$;

T — the time interval between two expected departure times between t_{i-1} and t_i, which is called valid request time, $\mathrm{T} = t_i - t_{i-1}$.

When solving the model, to ensure α and T are not extremely big and extremely small, the values of α and T were given constraints.

$$\alpha_{min} \leq \alpha \leq \alpha_{max}$$
$$T_{min} \leq T \leq T_{max}$$

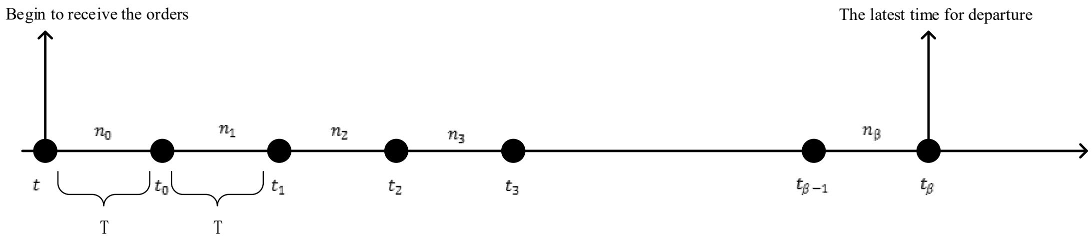

Fig.1 diagram of bus departure

At any time after the bus receiving orders, once the number of passengers is up to N or the delaying departure time reaches the threshold, the bus departs.

Every morning at a fixed time, the first bus begins to accept orders. In the meanwhile, the initial bus fare is X_0 and the expected departure time is t_0. At any time before t_0, once the reservation number reaches N, the bus can depart; otherwise, if after the first valid request time, T, namely t_0 moment, the total orders are still less than N, the departure time is pushed to $t_1(= t_0 + T)$. At the moment, there are two choices for passengers: to cancel the order or continue to wait. For the passengers continuing to wait, their fare would be discounted to αX_0.

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At t_β, the bus departs regardless of the number of orders. And the fares of different passengers: (1)for the passengers arrive during $t \sim t_0$, the fare is $\alpha^\beta X_0$, (2) for those arrive during $t_0 \sim t_1$, the fare is $\alpha^\beta X_0$, …, (β) for those arrive during $t_{\beta-2} \sim t_{\beta-1}$, the fare is αX_0, (β+1) for those arrive during $t_{\beta-2} \sim t_{\beta-1}$, the fare is X_0.

2.4 The cost of a single bus (y) and initial bus fare ($\boldsymbol{X_0}$)

The cost of single bus, y, is determined by bus operation cost and the fixed cost. The fixed cost includes share equally of buses purchasing, personnel wages, etc.

$$y = mL + m_0 \tag{1}$$

where: y — the cost of a single bus running

m — cost of unit mileage when bus running

L — running mileage of the origin point and the destination point

m_0 — constant, consisting of buses purchasing share, personnel wages, etc.

An expected profit rate, γ, and a minimum guaranteed profit rate, ρ, are defined. The expected profit rate is the profit rate expected by the carrier. The minimum guaranteed profit rate is the profit rate that the carrier can get regardless of the fare discount, which is guaranteed.

The initial bus fare, X_0 ,is determined by the expected profit rate, the bus capacity and the cost of the bus. That is

$$X_0 = (1 + \gamma)\frac{(mL+m_0)}{N} \tag{2}$$

According to the minimum guaranteed profit rate, ρ, there is following constraint

$$\alpha^k(1+\gamma) \geq (1+\rho) \qquad (3)$$

2.5 The relationship between the passengers' waiting time and waiting behaviour

During the valid request time, T, there will be passengers who do not want to wait and choose to cancel the reservations and others who are willing to wait until the next expected departure time because of the fare discount.
The probability of a passenger not cancelling reservation halfway is assumed to obey exponential distribution.

$$f(x)=\begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases} \qquad (4)$$

Where: λ is the parameter of the exponential distribution and its value can be determined by the investigation of the waiting passengers' psychology and be assumed.

2.6 Single bus income model and passenger's waiting time model

The single bus profit is the difference between the single bus income and the cost. The single bus income is equal to the sum of products of the number of passengers arriving at different moments and fares of different moments. According to the fare compensation mechanism, the bus fares are different for the passengers arriving at different time.
So, the single bus profit model is

$$z_1 = \Big\{\mathrm{floor}\Big[\big(n_0 - \sum_{i=1}^{k-1} n'_{0i}\big) \cdot \int_0^{\mathrm{T}} f(T)dT\Big] \cdot \alpha^k X_0 + \mathrm{floor}\Big[\big(n_1 - \sum_{i=2}^{k-1} n'_{1i}\big) \cdot \int_0^{\mathrm{T}} f(T)dT\Big] \cdot \alpha^{k-1} X_0 + \cdots + \mathrm{floor}\Big[\big(n_{k-2} - n'_{k-2\,k-1}\big) \cdot \int_0^{\mathrm{T}} f(T)dT\Big] \cdot \alpha^2 X_0 + \mathrm{floor}\Big[n_{k-1} \cdot \int_0^{\mathrm{T}} f(T)dT\Big] \cdot \alpha X_0 + n_k X_0\Big\} - (\mathrm{mL} + m_0) \qquad (5)$$

The passenger's total waiting time, $T_{\mathrm{sum\ i}}$, is the sum of the waiting time of passengers on the bus after departure (that is, the waiting time of passengers leaving halfway is not in consideration). The total waiting time at t_k:

$$z_2 = \big(n_0 - \sum_{i=1}^{k} n'_{0i}\big) \cdot t_k - \sum_{i=1}^{\left(n_0 - \sum_{j=1}^{k} n'_{0j}\right)} t_{0i} + \big(n_1 - \sum_{i=2}^{k} n'_{1i}\big) \cdot t_k - \sum_{i=1}^{\left(n_1 - \sum_{j=2}^{k} n'_{1j}\right)} t_{1i} + \big(n_2 - \sum_{i=3}^{k} n'_{2i}\big) \cdot t_k - \sum_{i=1}^{\left(n_2 - \sum_{j=3}^{k} n'_{2j}\right)} t_{2i} + \cdots + \big(n_{k-2} - \sum_{i=k-1}^{k} n'_{(k-2)\,i}\big) \cdot t_k - \sum_{i=1}^{\left(n_k - \sum_{j=k-1}^{k} n'_{(k-2)\,j}\right)} t_{(k-2)\,i} + \big(n_{k-1} - n'_{(k-1)\,k}\big) \cdot t_k - \sum_{i=1}^{\left(n_{k-1} - n'_{(k-1)\,k}\right)} t_{(k-1)\,i} + n_k \cdot t_k - \sum_{i=1}^{n_k} t_{ki} \qquad (6)$$

Above all, we built a multi-objective optimization model:

$$max z_1 = \Big\{\mathrm{floor}\Big[\big(n_0 - \sum_{i=1}^{k-1} n'_{0i}\big) \cdot \int_0^{\mathrm{T}} f(T)dT\Big] \cdot \alpha^k X_0 + \mathrm{floor}\Big[\big(n_1 - \sum_{i=2}^{k-1} n'_{1i}\big) \cdot \int_0^{\mathrm{T}} f(T)dT\Big] \cdot \alpha^{k-1} X_0 + \cdots + \mathrm{floor}\Big[\big(n_{k-2} - n'_{k-2\,k-1}\big) \cdot \int_0^{\mathrm{T}} f(T)dT\Big] \cdot \alpha^2 X_0 + \mathrm{floor}\Big[n_{k-1} \cdot \int_0^{\mathrm{T}} f(T)dT\Big] \cdot \alpha X_0 + n_k X_0\Big\} - (\mathrm{mL} + m_0) \qquad (7)$$

$$\min z_2 = \left(n_0 - \sum_{i=1}^{k} n'_{0i}\right) \cdot t_k - \sum_{i=1}^{\left(n_0 - \sum_{j=1}^{k} n'_{0j}\right)} t_{0i} + \left(n_1 - \sum_{i=2}^{k} n'_{1i}\right) \cdot t_k - \sum_{i=1}^{\left(n_1 - \sum_{j=2}^{k} n'_{1j}\right)} t_{1i} + \left(n_2 - \sum_{i=3}^{k} n'_{2i}\right) \cdot t_k - \sum_{i=1}^{\left(n_2 - \sum_{j=3}^{k} n'_{2j}\right)} t_{2i} + \cdots + \left(n_{k-2} - \sum_{i=k-1}^{k} n'_{(k-2)\,i}\right) \cdot t_k - \sum_{i=1}^{\left(n_k - \sum_{j=k-1}^{k} n'_{(k-2)\,j}\right)} t_{(k-2)\,i} + \left(n_{k-1} - n'_{(k-1)\,k}\right) \cdot t_k - \sum_{i=1}^{\left(n_{k-1} - n'_{(k-1)\,k}\right)} t_{(k-1)\,i} + n_k \cdot t_k - \sum_{i=1}^{n_k} t_{ki}$$
(8)

S.T.

$$\left(n_0 - \sum_{i=1}^{k} n'_{0i}\right) \geq 0 \quad (9)$$

$$\left(n_1 - \sum_{i=2}^{k} n'_{1i}\right) \geq 0 \quad (10)$$

… … …

$$\left(n_{k-2} - \sum_{i=k-1}^{k} n'_{k-2\,i}\right) \geq 0 \quad (11)$$

$$(n_{k-1} - n'_{k-1\,k}) \geq 0 \quad (12)$$

$$\alpha^k (1 + \gamma) \geq (1 + \rho) \quad (13)$$

$$X_0 > 0 \quad (14)$$

$$n_i \geq 0 \quad (15)$$

$$\mathrm{T} = t_i - t_{i-1} \quad (16)$$

$$n'_{0j} \geq 0 \quad (17)$$

$$n'_{ij} \geq 0 \quad (18)$$

$$t_\beta = t + kT \quad (19)$$

$$\alpha_{min} \leq \alpha \leq \alpha_{max} \quad (20)$$

$$T_{min} \leq \mathrm{T} \leq T_{max} \quad (21)$$

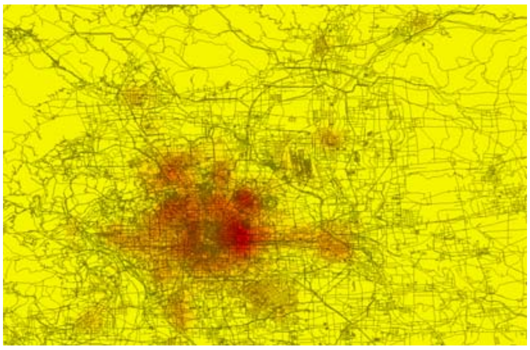
3 Data processing

The data of this paper came from the samples of DIDI's orders. The data of the period of 7:00:00~7:40:00 on December 22, 2016, in Beijing, was analyzed using the kernel density estimation based on the travel distance. The principle is as follow:

$$f_h(x) = \frac{1}{n} \sum_{i=1}^{n} K_h\,(x - x_i) = \frac{1}{nh} \sum_{i=1}^{n} K_h\,(\frac{x - x_i}{h}) \quad (22)$$

$K_h(x)$ is the kernel function[10]. There are many kinds of kernel functions, such as Gaussian, uniform, triangular, Biweight, triweight, Epanechnikov, normal and so on. *h* is a smooth parameter, called bandwidth (bandwidth), also called a window. This paper uses the Gaussian kernel function and the default optimal bandwidth to estimate the kernel density.

ArcGIS was used to show the distribution of the starting points and the ending points. It can be seen that most of the the starting points and ending points are concentrated in the central city and a few of them are located in Tongzhou, airports and other places far away from the central city.



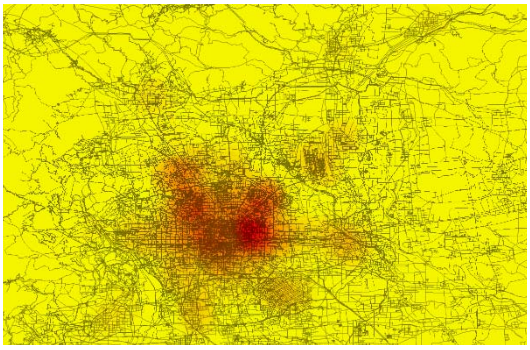


Fig.4 The distribution of starting points Fig.5 The distribution of ending points

The model established in this paper is based on the following characteristics: the starting points and the ending points are within the same small radius. They occurred in the areas where trips are not too dense. Therefore, according to the characteristics of the model, the data whose travel distance was larger than 25000m were selected and 79 sets of data were obtained. Then, the 79 sets of data were used to test the model.

4 Model solution

We choose Yizhuang Development Zone and Tongzhou District in Beijing as OD points. The parameters assignment are shown in the following table.

Tab.1 Parameters assignment

parameters	parameter values		
N	20	λ	600
X_0	17.4	γ	1
t	7:00:00	ρ	0.5
y	174	T_{min}	300
m	4	T_{max}	600
L	36	α_{min}	0.8
m_0	30	α_{max}	0.95
		k	3

From the Tongji Nan Road subway station in Yizhuang Development Zone to the Tongzhou District government, through the East sixth ring road, the travel distance is about 35.7km, so the running mileage, L, is assigned as 36km. According to Tab.1, it can be calculated that the single bus cost is 174 yuan. We select the minibus whose capacity is 20 and make the expected profit margin 1 and minimum guaranteed profit margin 0.5. From the above, the original bus fare is 17.4 yuan. The operating bus begins to accept requests at 7:00:00, and the latest time for departure is to be 40 minutes after accepting orders, that is k=3. We assign

λ to 600, and to ensure the profit of the operation company, we assign γ to 1 and ρ to 0.5. At the same time, to ensure α and T are not extremely big and extremely small, the T is restrained between 300 and 600 and α between 0.8 and 0.95.

According to the parameters assignment, LINGO was employed to solve the model. LINGO is the best choice for optimization model. For multi-objective optimization, the common methods include main objective, linear weighted summation, exponential weighted product, etc. In this paper, the linear weighted summation method is used. ω_1 is the weight of bus profit function, and ω_2 is the weight of waiting time function. The following results are obtained by the different values of ω_1 and ω_2.

Tab.2 The optimal solutions of the model with different weights

ω_1	ω_2	α	T(s)	$\max z_1$ (yuan)	$\min z_2(s)$	$\bar{T}(min)$	departure time
0.7	0.3	0.95	305.0895	96.1183	11671.8	9.73	7:20:20
0.75	0.25	0.95	419.0728	133.037	20790.42	17.33	7:27:56
0.8	0.2	0.95	466.5133	140.037	24585.07	20.49	7:31:06

From Tab.2, with the two weights changing, the optimal solutions of the model change. With the increase of the weight of the first objective, the bus profit gradually increases and the total waiting time decreases. Meanwhile, valid requests time gradually increases with ω_1 increasing, which means the waiting time of passengers choosing to wait increases, conforming to the actual condition. However, the optimal solution of fare discount, α, remains unchanged, which means relevant parameters should be adjusted.

Transfer times, sites number, travel time and travel cost of different travel methods are compared in the following table.

Tab 3 The comparison of different transportation

transportation	Transfer times	Sites number	Travel time	Travel cost
subway	1	28	1h and 28min	9 yuan
Traditional bus	1	27	1h and 40min	7 yuan
taxi	--	--	50min	93 yuan
On-demand bus	--	--	1h and 10min	17.4 yuan

According to Tab 3, from Yizhuang to Tongzhou, whatever by bus or subway, it is a travel of long-time and many-transfers. If choosing the taxi, the travel cost is too high to afford for most passengers. Therefore, the on-demand bus may be a better alternative.

5 Conclusion

In this paper, the mechanism of bus fare compensation and the mechanism of reservation cancelling are put forward. Based on the two mechanisms, the on-demand bus model is built and tested by the data from Didi. According to the results, the travel time is shorter than traditional bus and the cost is lower than the taxi.

The paper explores an on-demand bus considering the passengers' waiting time. The on-demand bus can make full use of public transportation resources and improve the bus service level to a certain extent. It provides a theoretical basis for the establishment of the on-demand bus systems.

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